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MCR-003-001544 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

May / June - 2018

S - 503 : Statistics Inference

(New Course)

Faculty Code : 003
Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Question No. 1 carries 20 marks.
(2) Question No. 2 and 3 each carries 25 marks.
(3) Students can use their own scientific calculator.

1 Filling the blanks and give short answers : **20**
(Each 1 mark)

(1) Define Parameter space.

(2) Name different criteria of good estimators.

(3) Write likelihood function of $f(x,\theta)=\theta(1-\theta)^{x-1}$

(4) Write likelihood function of

$$f(x,\theta)=\binom{-k}{x}\theta^{k}(\theta-1)^{x};0\le\theta\le 1.$$

(5) Obtain Cramer-Rao lower bound of variance of unbiased estimator of parameter of $f(x,\theta)=\theta x^{\theta-1}$

(6) __________ is an unbiased estimator of p^2 in Binomial distribution.

(7) If $f(x;\theta)$ is a family of distributions and $h(x)$ is any statistic such that $E\left[h(x)\right] = 0$, then $f(x;\theta)$ is called __________.

(8) If $S = s(X_1, X_2, X_3, \ldots, X_n)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\theta)$ for $i = 1, 2, 3, \ldots, n$ can be factorised as $g(s,\theta)h(x)$, then $s(X_1, X_2, X_3, \ldots X_n)$ is a __________.

(9) If a random sample $x_1, x_2, x_3, \ldots, x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of μ is __________.

(10) Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from a density $f(x,\theta) = \theta e^{-\theta x}$. Then the Cramer-Rao lover bound of variance of unbiased estimator is __________.

(11) If $T_n = t_n(X_1, X_2, X_3, \ldots X_n)$, an estimator of $\tau(\theta)$, is such that $\lim_{n \to \infty} \left[T_n - \tau(\theta)\right]^2 = 0, T_n$ is said to be __________ consistent.

(12) A sample constant representing a population parameter is known as __________.

(13) If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to __________.

(14) For a rectangular distribution $\frac{1}{(\beta-\alpha)}$, the maximum likelihood estimates of α and β are __________ and __________ respectively.

(15) If $x_1, x_2, x_3, \ldots x_n$ is a random sample from an infinite population and S^2 is defined as $\frac{\sum\left(x_i-\bar{x}\right)^2}{n}, \frac{n}{n-1}S^2$ is an __________ estimator of population variance σ^2.

(16) The estimator of σ^2 based on random sample $x_1, x_2, x_3, \ldots, x_n$ from a population $N\left(\mu, \sigma^2\right)$ by method of moments is __________.

(17) If T_n is an estimator of a parameter θ of the density $f(x;\theta)$ the quantity $E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2$ is called the __________.

(18) A single value of an estimator for a population parameter θ is called its __________ estimate.

(19) Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is __________.

(20) The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is __________.

2 (A) Write the answer any **three** : (Each 2 marks) **6**

(1) Define Sufficiency.

(2) Define Most Powerful Test (MP test).

(3) Show that sample mean is more efficient than sample median for Normal distribution.

(4) Define Complete family of distribution.

(5) Obtain an sufficient estimator of θ by for the following distribution

$$f(x;\theta) = \theta^x (1-\theta)^{(1-x)} ; x = 0,1$$

(6) Obtain an unbiased estimator of θ by for the following distribution

$$f(x;\theta) = \frac{1}{\theta} e^{-\left(\frac{x}{\theta}\right)} ; 0 \le x \le \infty, \theta > 0$$

(B) Write the answer any **three** : (Each 3 marks) **9**

(1) Show that $\frac{x(x-1)}{n(n-1)}$ is an unbiased estimator of p^2 in Binomial distribution.

(2) Obtain MLE of parameter p for the following distribution : $f(x; p) = pq^x ; x = 0,1,2,\ldots\infty$

(3) Let $x_1, x_2, x_3, \ldots, x_n$ be random sample taken from $N(\mu, \sigma^2)$ then find sufficient estimator of μ and σ^2.

(4) Obtain an unbiased estimator of population mean of $\aleph^2$ distribution.

(5) If A is more efficience than B then prove that $Var(A)+Var(B-1)=Var(B)$.

(6) Use the Neyman Pearson lemma to obtain the best critical region for testing $H_0:\lambda=\lambda_0$ against $H_1:\lambda=\lambda_1$ in the case of of Poisson distribution with parameter λ.

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) State Neyman-Pearson lemma and prove it.

(2) Obtain MVBE of σ^2 for Normal distribution.

(3) For the double Poisson distribution

$$P(X=x)-\frac{1}{2}\frac{e^{-m_1}{m_1}^x}{x!}+\frac{1}{2}\frac{e^{-m_2}{m_2}^x}{x!};0,1,2,\ldots.$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1'\pm\sqrt{\mu_2'-\mu_1'-(\mu_1')^2}$

(4) Construct SPRT of Poisson distribution for testing $H_0:\lambda=\lambda_0$ against $H_1:\lambda=\lambda_1(>\lambda_0)$.

(5) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2,σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination?

3 (A) Write the answer any **three** : (Each 2 marks) **6**

(1) Show that $\sum x_i$ is a sufficient estimator of θ for Geometric distribution.

(2) Define Uniformly Most Powerful Test (UMP test).

(3) Define ASN function of SPRT.

(4) Define Unbiasedness.

(5) Define Efficiency.

(6) Obtain likelihood function of Negative Binomial distribution.

(B) Write the answer any **three** : (Each 3 marks) **9**

(1) Obtain unbiased estimator of $\frac{kq}{p}$ of Negative Binomial distribution.

(2) Prove that $E\left(\frac{\partial logL}{\partial\theta}\right)^2 = -E\left(\frac{\partial \log L}{\partial\theta^2}\right)$.

(3) Obtain estimator of θ by method of moments in the following distribution

$$f(x;\theta) = \frac{1}{\theta}e^{-\left(\frac{x}{\theta}\right)}\ ; 0 \le x \le \infty, \theta > 0$$

(4) Obtain MVUE of parameter θ for Poisson distribution.

(5) Give a random sample $x_1, x_2, x_3, \ldots, x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta.$ Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x; x \ge 0.8\}$.

(6) Obtain Operating Characteristic (OC) function of SPRT.

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) State Crammer-Rao inequality and prove it.

(2) Give a random sample $x_1, x_2, x_3, \ldots, x_n$ from distribution with p.d.f.

$$f(x;\theta) = \theta e^{-\theta x} \; ; 0 \le x \le \infty, \theta > 0$$

Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(3) Estimate α and β in the case of Gamma distribution by the method of moments

$$f(x;\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma\beta} e^{-ax} x^{\beta-1}; x \ge 0, \alpha \ge 0$$

(4) Construct SPRT of Binomial distribution for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.

(5) Obtain Likelihood Ration Test:

Let $x_1, x_2, x_3, \ldots, x_n$ random sample taken from $N(\mu, \sigma^2)$. To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \ne \sigma_0^2$